

## BASIC GEOMETRY CONCEPTS AND DEFINITIONS:

**Geometry:** It is an area of knowledge which studies any elements and operations on/in the plane such as points, lines or shapes. It comes from the Greek geo "earth", metry "measurement".

**Point:** In geometry a point can be defined as the place or location where two lines intersect. A point has no dimensions, no height and no width.

**Line:** a one-dimensional object formed of infinite points . It has no end points and continues on forever in a plane.

**Ray:** A line which begins at a particular point (called the endpoint) and extends endlessly in one direction.

**End point:**An End Point is a point at which a line segment or a ray ends or starts.

**Midpoint:**It is the point that is halfway between the endpoints of the line segment.

**Line segment:** It is a line with two endpoints

**Straight line:** A line whose points follow the same direction.

**Plane:** It is a two-dimensional (height and width) surface. In the space a plane can be defined by two parallel lines, two intersecting lines or one point and a straight line.

**Length:**Measurement of something from end to end

Listen and watch this video about basic geometry language online

<http://youtu.be/iI0EJrY64qE>



AB are End points for the line segment, C is its midpoint. D is the Ray's Endpoint.

## GEOMETRY DRAWING AND SUPPLIES CONCEPTS AND DEFINITIONS:

**Freehand:** Drawn by hand without guiding instruments, measurements.

**Line/Technical drawing:** It is a drawing made with the help of supplies. It is usually the kind of drawing used for architecture or engineering plans.

**Compass:** It is a tool for drawing circles and arcs and also for measuring distances between points, consisting of two arms linked by a hinge.

**Protractor:** an instrument for measuring or drawing angles on paper, usually a flat semicircular transparent plastic sheet graduated in degrees

**Set squares (UK) Triangles (US):** They are two special rulers with a triangular shape. One is called the **45° triangle (45°square, UK)** and the other the **30/60° triangle**. Both have the 90° angle. So, they are used to draw certain angles and also to draw parallel lines.

**Eraser (Rubber, UK):**an object, such as a piece of rubber, used for deleting something.

**Marker (Felt-tip pen, UK):** a pen having a writing point made from pressed fibres.

**Ruler:** An instrument used to draw straight lines. Also called straight edge

### COMPLETE THE FOLLOWING SENTENCES:

- 1- I need a .....to draw a perfect circle.
- 2- If you don't use the..... you won't get perfect parallels.
- 3- See my ..... drawing!!! now i'm going to use instruments to draw it more accurately
- 4- If you use a pencil, you can use an..... to delete the lines.
- 5- Have you seen my .....? It is a plan to build a house.

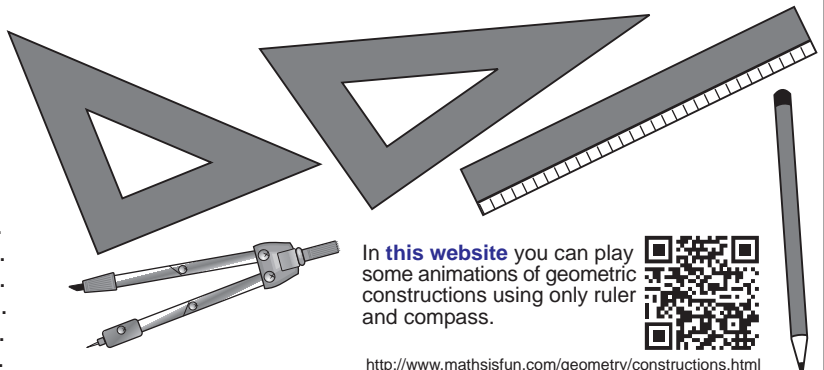
There are two types of main geometries: Planar Geometry, which only studies flat objects (two dimensional) such as points, lines and shapes; And Descriptive Geometry that studies the depiction of three dimensional objects like polihedra or other shapes or solids with volume.

For studying and practising planar geometry we will use the supplies shown below. But some people only uses a compass and a ruler and that is called "straight edge geometry".

There is also a "rusty compass" geometry that never changes the radius of the compass for its constructions, and Mascheroni demonstrated that geometry can be made only with a compass without a ruler.

### WHAT WOULD YOU USE TO DRAW...:

- 1- Perpendicular lines?.....
- 2- A set of concentric circles?.....
- 3- A long straight line?.....
- 4- A circle?.....
- 5- A few paralel lines?.....
- 6- A portrait?.....
- 7- An arc?.....
- 8- A landscape?.....
- 9- Certain angles?.....
- 10- copying lengths?.....



In [this website](http://www.mathsisfun.com/geometry/constructions.html) you can play some animations of geometric constructions using only ruler and compass.



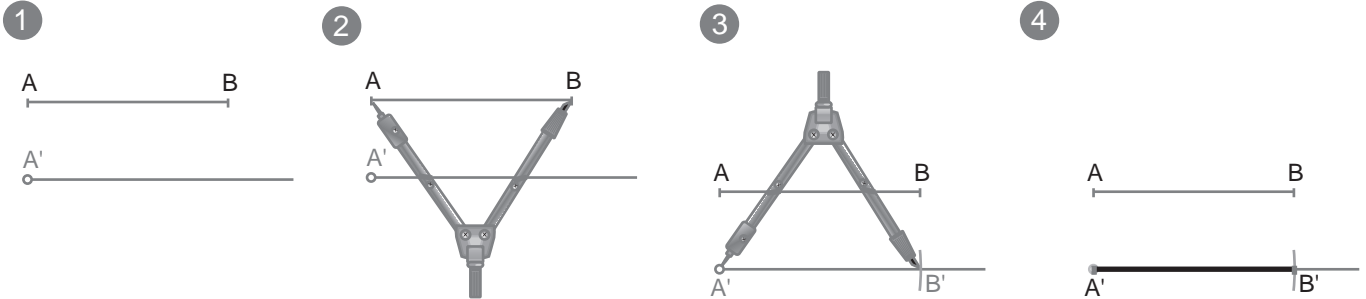
<http://www.mathsisfun.com/geometry/constructions.html>

To perform operations (additions or subtractions) with line segments we always use the compass to take lengths, to copy or move them. A ruler must be used to make the straight lines, while the compass will be the tool that works for giving lengths to the segments.

**LINE SEGMENT COPYING:** Given the segment AB, copy it with the same length.

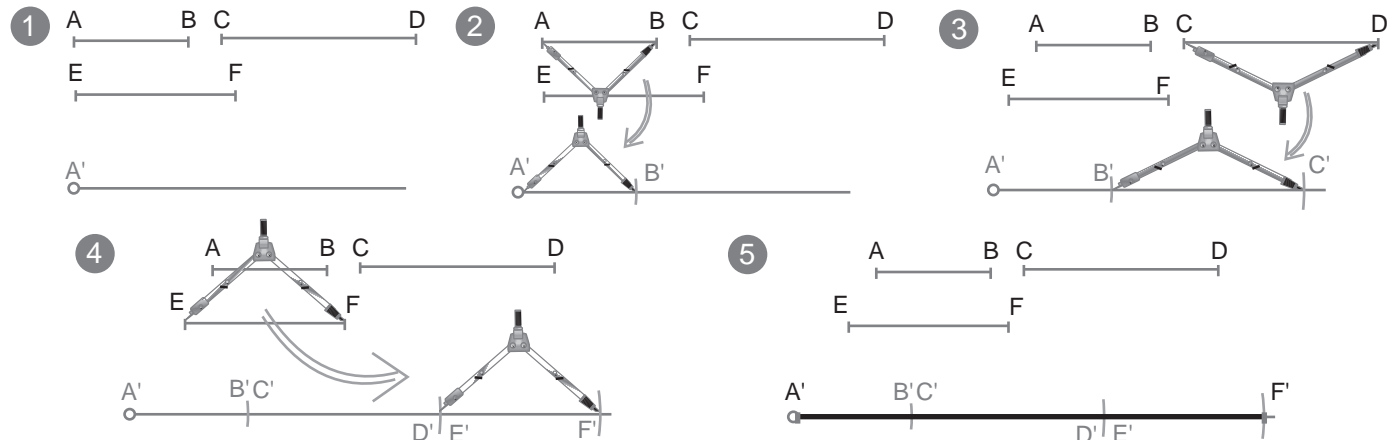
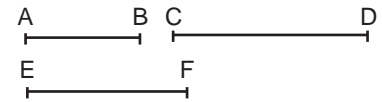


- 1st- Draw a ray from point A'.
- 2nd-With the compass take the length AB.
- 3rd- Take the length AB, kept with the compass. Set the center on the ray's point A' and draw an arc obtaining B'.
- 4th- Finally enhance with black ink the result (Important).



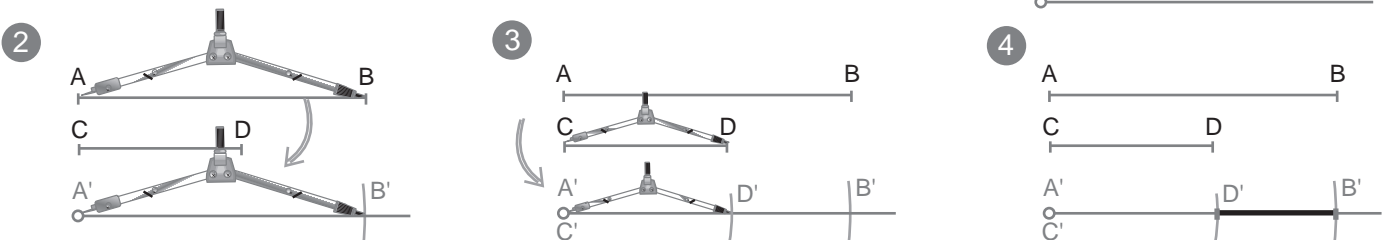
**LINE SEGMENTS ADDITION:** Given the line segments AB, CD and EF, add them graphically.

- 1st- Draw a ray from the point A'.
- 2nd-With the compass take the length AB, and copy it on the ray from A', obtaining B'. (This is AB segment line copying)
- 3rd- From B' we repeat the operation with the following segment line to add (CD).
- 4th- In this case we need to add three segments to get the addition. we repeat the operation with the last one.
- 5th- The result is the total of the three segments copied one right next to the other. That is to say, A'F'. We enhance the result with black ink (IMPORTANT).



**LINE SEGMENTS SUBTRACTION:**  $AB - CD$ , subtract them graphically.

- 1st- Draw a ray from the point A'.
- 2nd-Take, with compass, the AB length, the longest one, and copy it onto the ray from A', obtaining B'. ( AB segment line copying)
- 3rd- From A' we repeat the same operation with the segment CD. That is to say, we will copy the smaller segment onto the longer one already copied.
- 4th- The difference of lengths, length from D' to B', is the result. We must enhance the result with black ink.

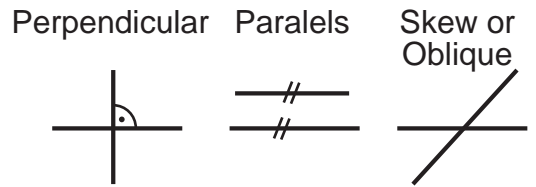


# IMPORTANT DEFINITIONS ABOUT ANGLES AND LINES:

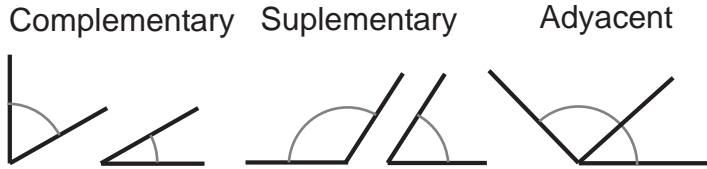
**Parallels:** They are lines which never intersect themselves, so all their points are equidistant.

**Perpendicular:** They are lines which meet forming four right angles.

**Oblique / Skew:** They are lines which are not parallels neither perpendicular.



**ANGLE:** It is a figure formed by two rays (sides of the angle) sharing a common endpoint (vertex).



**Complementary angles:** They are couples of angles which sum is  $90^\circ$ .

**Supplementary angles:** They are couple of angles which sum is  $180^\circ$ .

**Adjacent angles:** They are angles that share one side and the vertex.

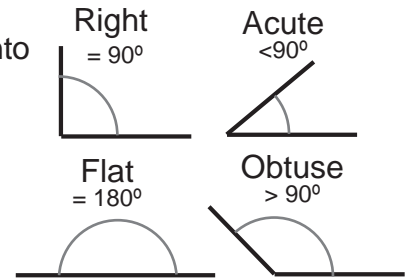
**Angle bisector:** It is the line segment or ray which divides an angle into two equal parts

**Right:** A right angle has 90 degrees.

**Obtuse:** A obtuse angle has more than 90 degrees.

**Acute:** An acute angle is the one that has less than 90 degrees.

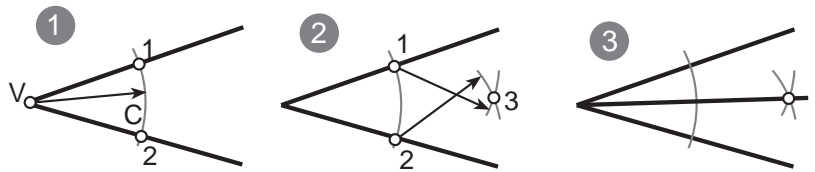
**Flat:** A flat angle has 180 degrees.



## The angle's bisector

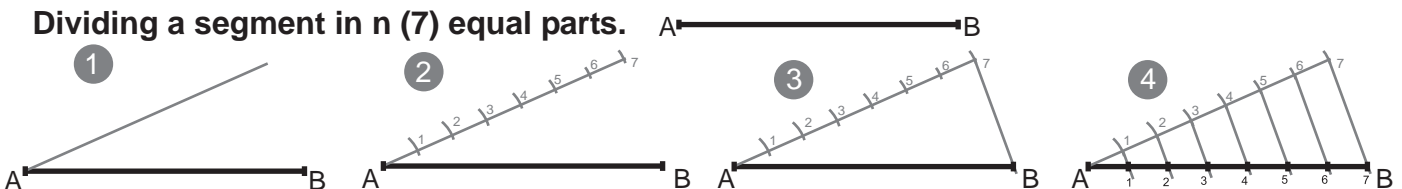
1st-With any radius and center on V vertex, draw an arc which intersects the rays of the angle in two points, 1 and 2.

2nd-Centered in these two points and a radius longer than half of the distance between them, draw two arcs which intersect in a point, 3.



3rd-Connect the point 3 with V vertex.

## Dividing a segment in n (7) equal parts.



1st- From A endpoint trace a ray forming any angle with the segment.

2nd-Using the compass, tracing arcs, (or a ruler) and with any length make seven equal parts (starting from the A endpoint) on the ray.

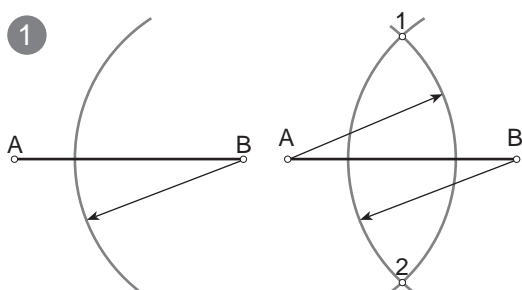
3rd-Connect the last division (7) with the other endpoint (B) of given segment. 4th-Draw parallels to 7-B segment through each division on the ray.

## Perpendicular segment line bisector:

Given the segment AB, Draw its perpendicular bisector.

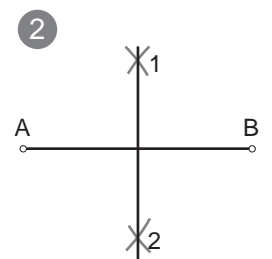
A line segment's perpendicular bisector is a perpendicular line through its midpoint. It can also be defined as "A set of points that are equidistant from the two endpoints of a line segment."

Construction:



1st- Draw two arcs with same radius centered in endpoints A and B. Two points, 1 and 2, are obtained where both arcs intersect.

2nd- Connect 1 with 2 to obtain the perpendicular line bisector. Enhance the result with black ink.

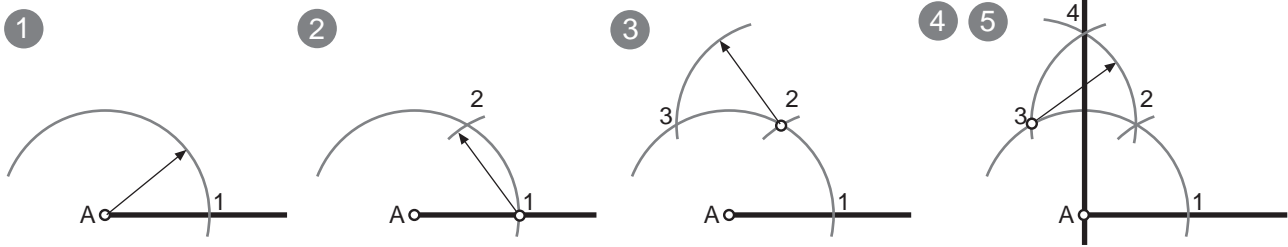


### Perpendicular line of a segment through an end point:

Ao \_\_\_\_\_

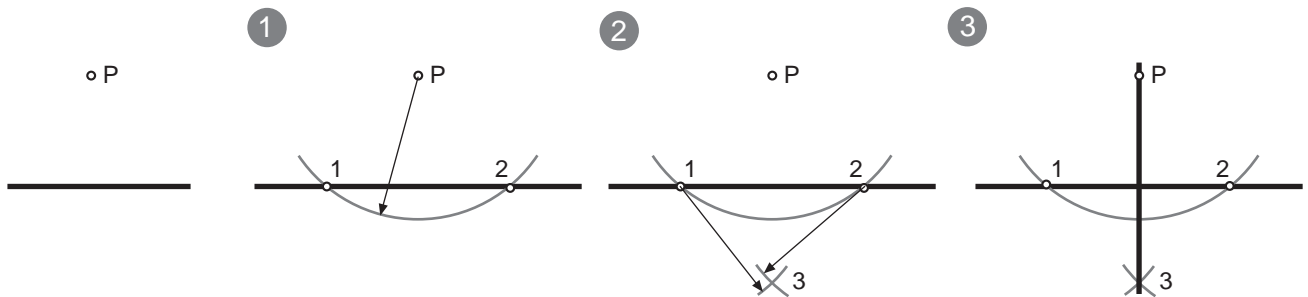
Given a line segment AB, draw a perpendicular line through its endpoint A.

- 1st-Draw an arc, with any radius, centered in A (close to half circle), this intersects the given line in point 1.
- 2nd-With the same radius, draw an arc centered in point 1, this intersects the first arc in point 2.
- 3rd-With the same radius, draw an arc centered in point 2, this arc intersects the first arc in point 3.
- 4th-With the same radius, draw an arc centered in point 3, this intersects the last arc in point 4.
- 5th-Connect points 4 and A. Draw the result with black ink.



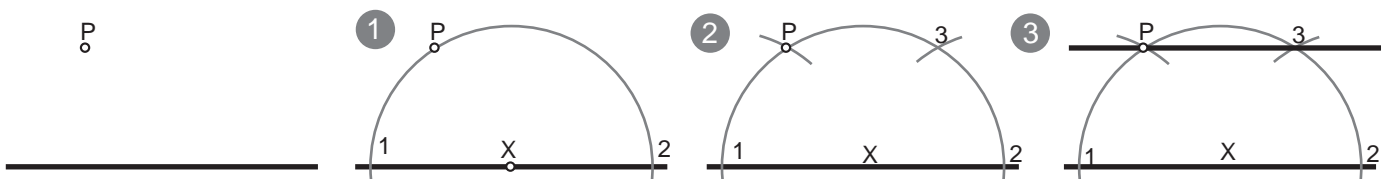
### Perpendicular line to another through an outer point of it:

- 1st-Draw an arc centered in the given point P intersecting the given line in points 1 and 2.
- 2nd-Draw two arcs with same radius centered in 1 and 2. The two arcs intersect in point 3.
- 3rd-Connect 3 with the given point. Draw the result with black ink.



### Parallel line to another one through an outer point of the given one:

- 1st- Draw half circle, through the given point P, with center on the given line. This intersects the given line in point 1 and 2.
- 2nd- with radius 1P draw an arc centered in point 2 intersecting the circle in point 3.
- 3rd- Connect 3 with P. Draw the result with black ink.

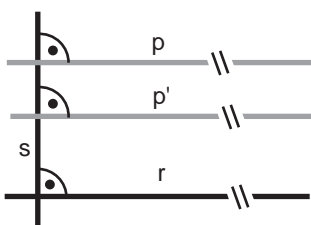


### Parallel to a given line at a given distance (d):

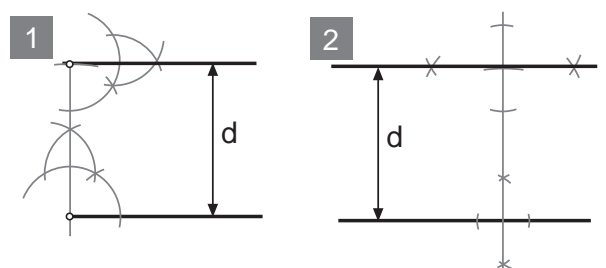
\_\_\_\_\_ d \_\_\_\_\_

The distance between two lines is the length of a line segment perpendicular to both given lines.

If we have a given line (r), and a perpendicular to it (s), any perpendicular (p) to s will be parallel to r.



Therefore we can use any of the "perpendicularity" methods to solve this problem. To the right we are showing two of them.



## ANGLES COPYING WITH RULER AND COMPASS:

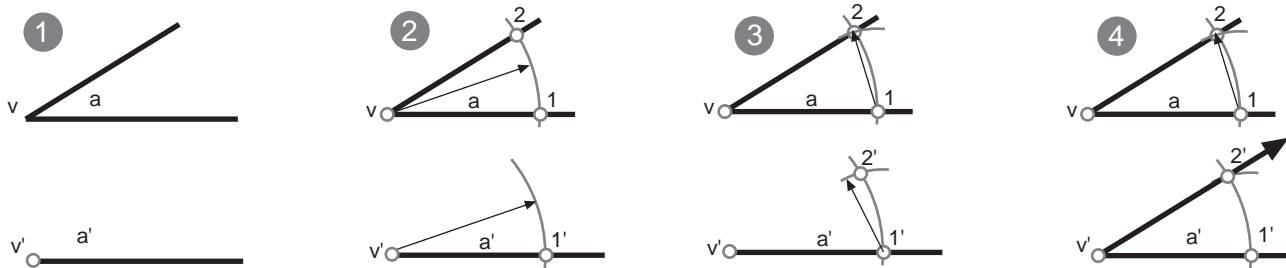
Given an angle (a) draw another (a') equal to the first one.

1st- Draw a ray and label the new vertex (the ray's endpoint) as v'.

2nd- Draw an arc centered in v cutting the two rays of an angle in 1 and 2. Centered in v', draw another arc with the same radius to the first one intersecting the ray in 1'.

3rd- Draw an arc centered in 1' with a radius 1-2 intersecting the first arc in point 2'.

4th- Connect v' with 2'. Draw the result with black ink.



## ANGLES ADDITION WITH RULER AND COMPASS: Given the angles (a) and (b) draw another angle (c)=(a)+(b)

This is about copying one angle and copying the second one right on the upper ray of the first one, Both together are the result.

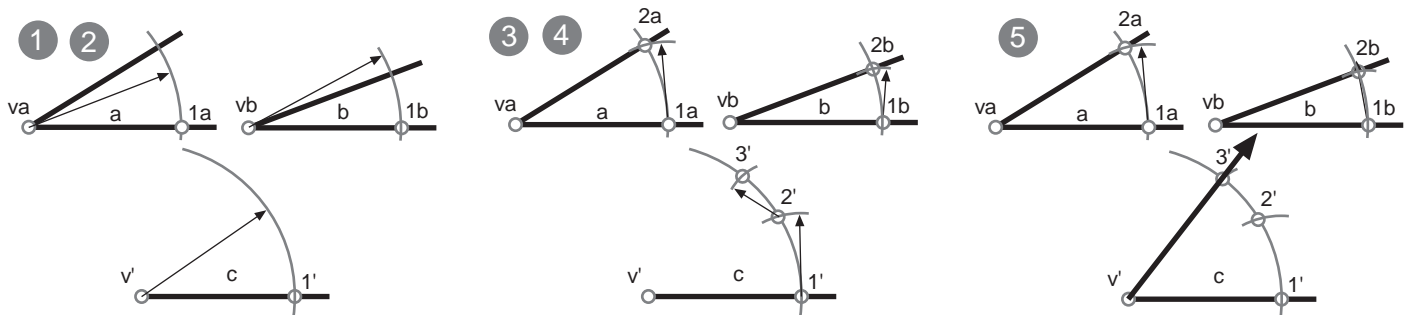
1st- Draw a ray and label the endpoint as v' which will be the vertex of the result a+b.

2nd- Draw arcs with the same radius centered in (va) y (vb) obtaining 1a and 1b. Draw another arc with the same radius centered in v' obtaining 1'.

3rd- with the compass take the length 1a-2a and draw an arc with that radius centered in 1' obtaining 2'.

4th- with the compass take the length 1b-2b and draw an arc with that radius centered in 2' obtaining 3'.

5th- Connect v' with 3'. label the resulting angle a+b=c with black ink.



## ANGLES SUBTRACTION WITH RULER AND COMPASS: Given the angles (a) and (b) Draw another (c) = (a-b)

Se This is about copying first the bigger angle and copying the smaller inside sharing one ray with the first one, the difference is the result.

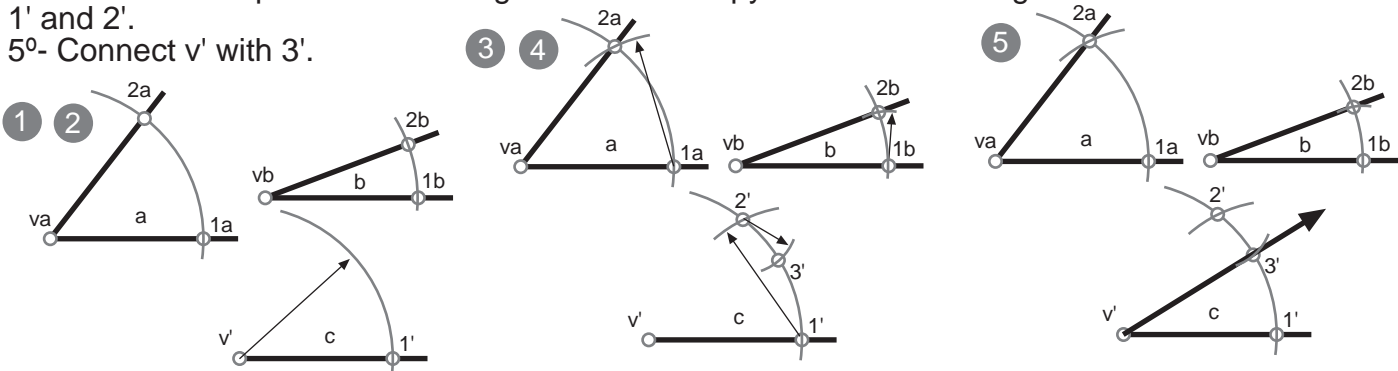
1st- Draw a ray and label one endpoint as V' which will be the vertex for the result.

2nd- Centered in (va) and (vb), draw further arcs with any radius but equal, these must intersect both rays of each angle in two points. Centered in v' trace an arc with the same radius obtaining on the v' ray the point 1'.

3rd- With the compass take the length 1a-2a and copy it from 1' obtaining 2'.

4th- With the compass take the length 1b-2b and copy it from 2' obtaining 3' that must be between 1' and 2'.

5th- Connect v' with 3'.





## IMPORTANT DEFINITIONS ABOUT CIRCLES:

**Circle:** Set of points at the same distance (equidistant) to a point called center.

**Circumference:** It is the full length of a circle. The complete distance around a circle.

**Center:** It is the point equidistant to any of the points on a circle.

**Radius:** It is the length of a line segment from the center to its perimeter. (plural Radii from latin or common english radiuses)

**Diameter:** A line segment that passes through the center and connects two points of a circle. its length is double than the radius.

**Chord:** A line segment which connects two points of the circle not through the center.

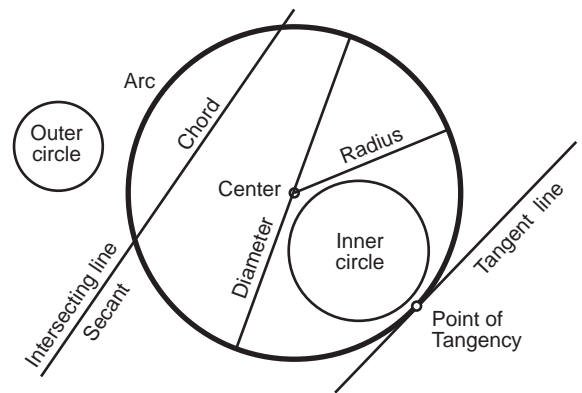
**Arc:** segment of a circle. Watch out!! ARCH is for architecture,

**Tangent:** Element (straight or curve lines) which touch a circle in one point.

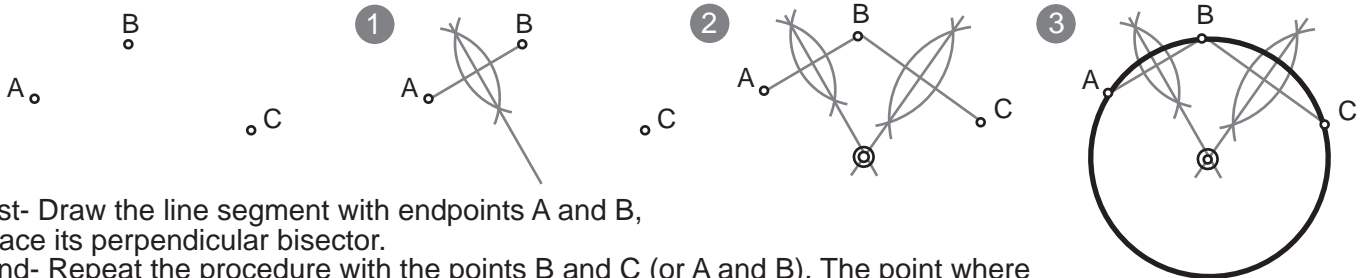
**Outer:** When an element (line or curve) doesn't touch the circumference, or another element.

**Inner:** When an element (line or curve) is fitted inside the circumference, or another element, being tangent or not.

**Concentric circles:** They are circles with different radius but placed in the plane sharing the center.



### Draw a circle through three given points (A, B and C)



1st- Draw the line segment with endpoints A and B, trace its perpendicular bisector.

2nd- Repeat the procedure with the points B and C (or A and B). The point where both perpendicular bisectors intersect is the center of the circle.

4th- With center in that point and radius to any of the given points, draw the circumference.

Knowing how to solve this problem we can easily find the center of a circle or an arc by drawing two chords of a circle and finding their perpendicular bisectors. The point where both bisectors intersect is the center of the circle.

Here you can see an [animation on how to find the center of an arc.](http://www.mathopenref.com/constcirccenter.html)



<http://www.mathopenref.com/constcirccenter.html>

### LANGUAGE. FALSE FRIENDS: CIRCLE-CIRCUNFERENCE



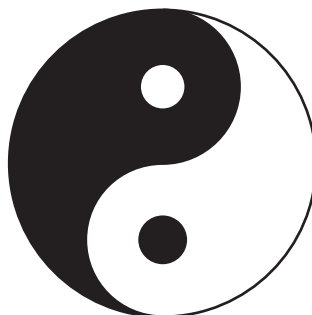
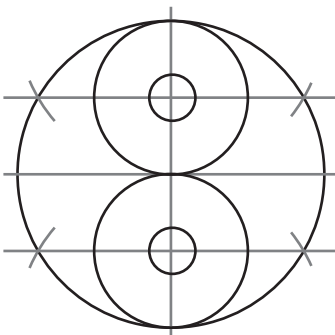
Watch [this video about language and notations of the circle](http://youtu.be/U2W7HPyC0cM) it will take you ten minutes.

<http://youtu.be/U2W7HPyC0cM>

"circunferencia" is translated into English as "circle"

"circumference" in English means "the perimeter of the circle"

There is no word in Spanish for "circumference" so in Spanish it is said "perímetro o longitud de la circunferencia". As well as there is no word for "círculo" in English which is called "area of a circle"

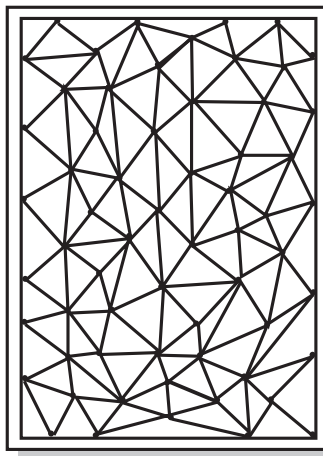
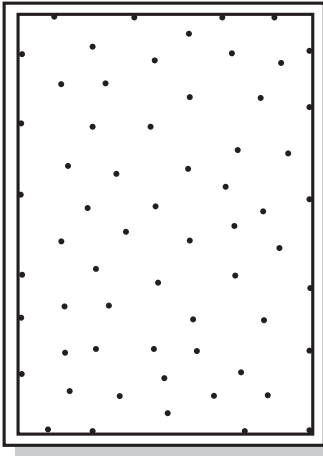


Bearing.

Source: <http://www.bearings-china.com/>



## TRIANGLES AND PARALLELS



1st- Draw dots spread all over the worksheet. Don't arrange them in a tidy way. **DO NOT** forget to draw some dots on the edges of the frame.

2nd- Connect each dot with the closer ones with a ruler.

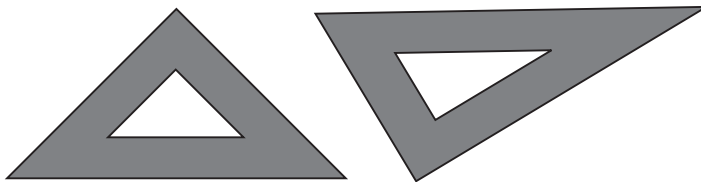
- The line segments connecting the dots cannot cross other line segments connecting other dots. If you let that happen the resulting triangles will be smaller than supposed.
- The line segments connecting dots cannot go through other dots.
- That is to say: each line segment only has two end points and no midpoints and does not intersect any other line segment.
- **DO NOT** forget to connect the dots on the edges with other dots.

3rd- If you follow correctly these two first steps you will have completed the worksheet with triangles.

4th- You must fill in with color markers each triangle with parallels using the set squares:

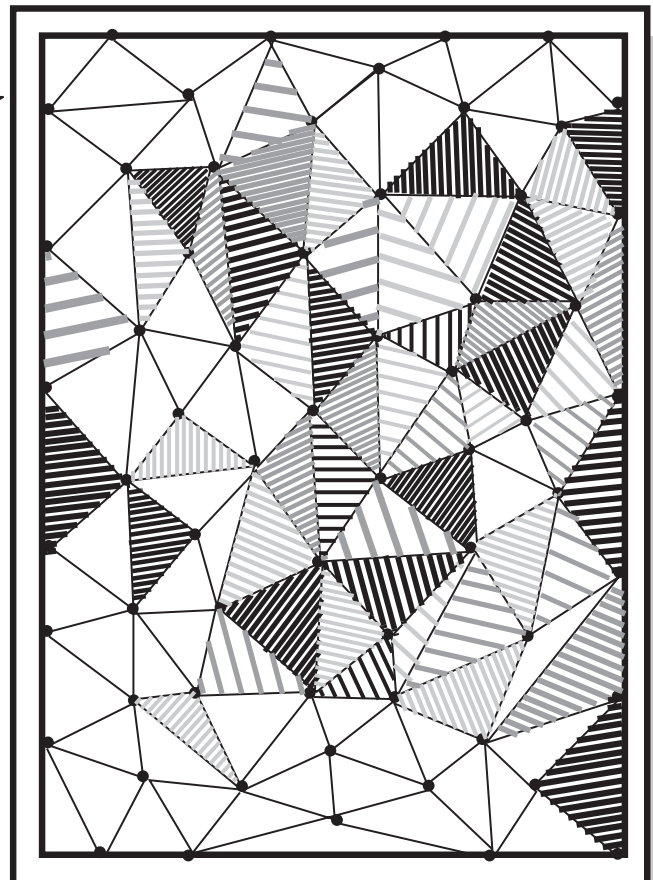
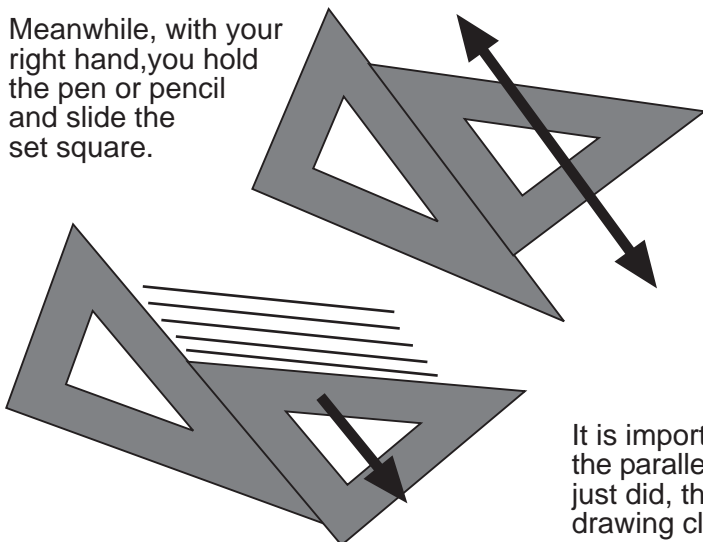
- **Triangles sharing sides or edges CANNOT contain the same color, neither the same slope or frequency of parallels. Each triangle only has to contain one color for its parallels.**
- You need to complete every triangle with parallels.
- It will always be better that you try to complete a design of something you think that can be abstract or figurative rather than assign colors randomly.

### USING THE SET SQUARES TO DRAW PARALLEL LINES



Use your "wrong hand" (left hand if you are right handed) to hold tight one of the set squares against the paper sheet. This set square must be static and cannot move under any concept. With that hand, with your index finger, hold the other set square as a break so it does not move while you draw the lines with your other hand.

Meanwhile, with your right hand, you hold the pen or pencil and slide the set square.

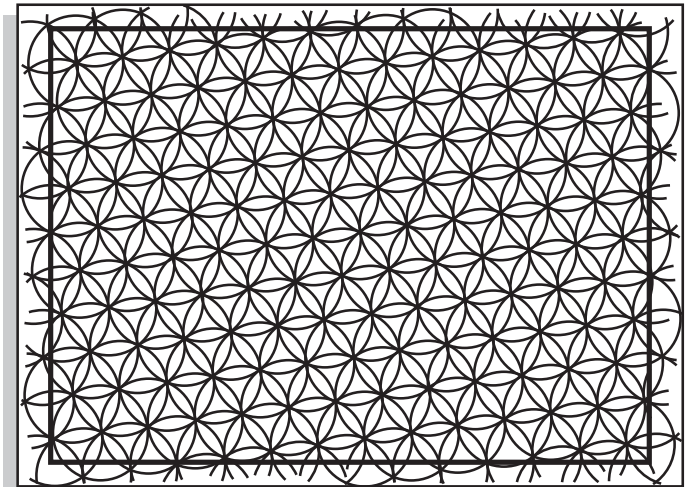
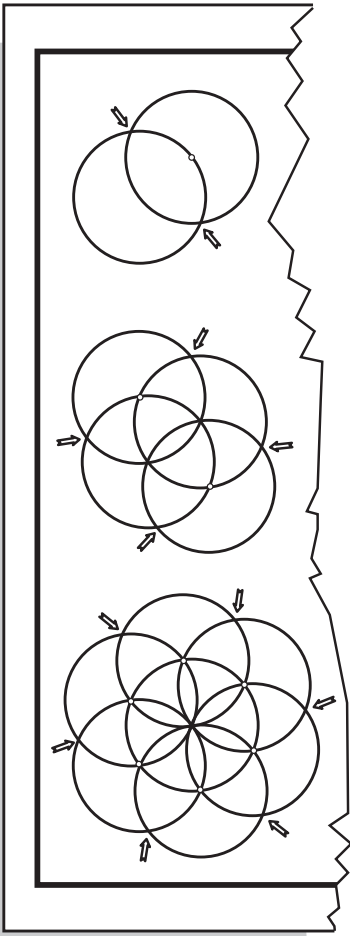


It is important that you slide the moving set square as you leave the parallel lines behind so you don't "step over" the parallels you just did, that way you will not mess up with fresh ink leaving the drawing clearer and neat.

# CIRCLES PATTERN 1

This worksheet is about filling in the drawing space with circles with the same radius (2,5 cm.). But you must do it following a simple pattern:

- 1st- Draw a circle (with a radius of 2,5 cm) centered in any place in the worksheet. It is better that you place it in the middle.
- 2nd- Draw another circle, with the same radius, centered in any of the points of the first circle you did.
- 3rd- The two intersecting points of both circles are new centers for new circles with the same radii. These will intersect in four new points to center new circles.
- 4th- As you keep on drawing circles you will obtain new intersecting points to center new circles. Remember all the circles must have the same radius!!
- 5th- Fill in all the worksheet. Draw also circles even out of the frame, since those might intersect and show you a center for a circle which part of it is inside the frame.
- 6th- Erase anything out of the frame.
- 7th- GIVE COLOR TO ALL THE WORKSHEET: IFollow a pattern for color. For instance, you can give the "petals" one or two colors (following a pattern) and do the same for the "curved triangles":



**IMPORTANT WARNING:** You must keep the lead of the compass very sharp. It is very important that you keep the same radius all the time and that you set the center accurately. If you have tried hard and couldn'tn make it try it with 3cm. radius circles.

## FLATLAND - Edwin A. Abbott 1838-1926

### 1. OF THE NATURE OF FLATLAND

I call our world Flatland, not because we call it so, but to make its nature clearer to you, my happy readers, who are privileged to live in Space.

Imagine a vast sheet of paper on which straight Lines, Triangles, Squares, Pentagons, Hexagons, and other figures, instead of remaining fixed in their places, move freely about, on or in the surface, but without the power of rising above or sinking below it, very much like shadows--only hard with luminous edges--and you will then have a pretty correct notion of my country and countrymen...

### 2. OF THE CLIMATE AND HOUSES IN FLATLAND

As with you, so also with us, there are four points of the compass North, South, East, and West. There being no sun nor other heavenly bodies, it is impossible for us to determine the North in the usual way; but we have a method of our own...

....Square and triangular houses are not allowed, and for this reason. The angles of a Square (and still more those of an equilateral Triangle,) being much more pointed than those of a Pentagon, and the lines of inanimate objects (such as houses) being dimmer than the lines of Men and Women, it follows that there is no little danger least the points of a square or triangular house residence might do serious injury to an inconsiderate or perhaps absentminded traveler suddenly running against them: and therefore, as early as the eleventh century of our era, triangular houses were universally forbidden by Law, the only exceptions being fortifications, powder-magazines, barracks, and other state buildings, which is not desirable that the general public should approach without circumspection...

Watch this [movie trailer of a cartoon version of flatland](http://youtu.be/714ZHwodLQA). It is subtitled in Spanish

